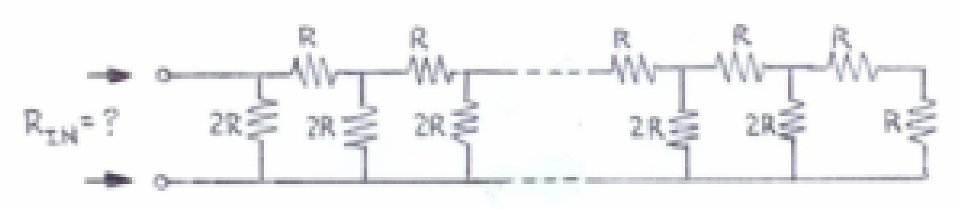
Circuit Challenge 2

Consider the circuit shown below.



The Situation

In your youth you went into a surplus store and purchased a box of about 200 carbon resistors. You got a good price, and then took them home. At home you noticed that about half the resistors are identically marked (same color code). The other half are identically marked with a different color code. You were not able to read the codes, but your neighbor had an ohm-meter. The result was that the resistors had the following two values: R and 2R.

Some time later (and after purchasing your own ohm-meter) you began constructing the circuit shown above. You soldered the resistors together, working from right-to-left. You ended up by placing the last 2R element shown at the left end of the circuit above. You had been measuring the resistance after the placement of each resistor, but you saw a pattern emerging, so you soldered rapidly until you had used up all your resistors. You measured the final input resistance and realized you could have achieved the same result with fewer resistors.

Question

Assuming there were 100 2R resistors and 101 R resistors, what is R_{IN}?

You may compare your R_{IN} with the actual R_{IN} by scrolling down to the analysis below.

Analysis for Circuit Challenge 2

R_{IN} = R

The circuit is known as an R-2R ladder. Starting with the right-most circuitry we have R+R in parallel with 2R, which is R. This R is in series with an R, making 2R. This new 2R is in parallel with 2R, which is R. We then proceed from right-to-left, until there is a final 2R in parallel with 2R, which is R.

Assume a voltage is applied at the terminals on the left. Assume the input current is i. This i "sees" 2R in parallel with 2R, so it divides into i/2 and i/2. One i/2 goes vertically down to the lower line, and then returns to the voltage source. The other i/2 proceeds to the right through the first horizontal R. It then "sees" 2R in parallel with 2R, so it divides into i/4 and i/4. This divide-by-2 (binary) pattern continues all the way to the nth loop. If, as in this challenge, there are 100 2R resistors, then the final current division would yield i_{100}, as follows:

$$i_n = i/(2^n)$$

Thus,

$$i_{100} = i/(2^{100})$$

Theoretically this is an attractive observation, but where, as here, n is a very large number, it cannot (by reason of resistor tolerances) be sustained beyond a single digit (perhaps 6, or 7, or 8). Why we might care about how large n can be is a discussion left to the world of A-to-D and D-to-A converters.

Want to Know More?

Here's one reference: http://en.wikipedia.org/wiki/Resistor_ladder