Circuit Challenge 4

Sometimes a practitioner of the electronics arts must reach beyond traditional circuit theory. This is one of those times.

A Late-night Conversation of Great Import

Suppose a worried group of astronomers had approached you a few hours ago. Their spokesperson said: "As we speak, a peculiar alignment of the planets with a certain comet is affecting the earth's rotational velocity. It will, in fact, gradually accelerate until mass becomes weightless at the equator. For now we are calling this (pause) *Condition Zero*. However, if we can generate ground wave pulses whose frequency of occurrence matches that of the earth's rotation at *Condition Zero*, and if we correctly direct those pulses at the equator, we will be able to negate the entire effect."

"I can appreciate that *Condition 0* will cause chaos at the equator," you replied. "What is it you want me to do?"

"Design the timer. We will use the output of your circuitry as the input to a rather massive transmission apparatus that has been hastily constructed and is now ready. Time is of the essence. Here is a phone number. Call it. Put audible 10 kHz bursts, 5-seconds wide, into the phone line, one burst at a time, at a rate that corresponds to the period of rotation associated with *Condition Zero*. These will be received by our anti-Condition-0 equipment. You must start the pulse string some time in the next three hours. Bursts delivered within +/-10% of the actual frequency will be adequate."

With that, they left. But there was a problem: They didn't give you the frequency! You called the number to get the frequency. The response was something akin to dialing a fax machine. You knew it was waiting for the first burst. The astronomers gave you no other number to call. What to do?

The Task Before You

Your basic plan is to discharge a 1 millifarad capacitor, force a DC current into it, then trip a comparator when there is 2.5V on the capacitor. Tripping the comparator will launch a 10KHz signal whose duration is 5 seconds. That signal will then stop. The capacitor will be discharged to 0V. The DC current will again flow into the capacitor. The ramp time for charging the capacitor will correspond to the time of 1 rotation of the earth at *Condition Zero*. Clearly, before you can complete the design of your circuit you must calculate the earth's period of rotation at an angular velocity that will cause weightlessness at the equator. Do that; then calculate the value of the required DC current source; then compare with what you find when you scroll down to the analysis.

Analysis for Challenge 4

Centrifugal force is given by

$$Force = mv^2/r$$

But force is also given by

$$Force = ma$$

where a is acceleration. For purposes of this discussion, a = g. Then we can write

$$mg = mv^2/r$$

We can cancel out the mass, and recognize that r must be the radius of the earth at the equator. r varies of course, but a quick look at Wikipedia indicates that the average radius of the earth is 3956.5 miles. g is about 32.2 ft/sec^2. Thus, we can write

$$32.2\ ft/sec^2 = v^2/(3956.5\ miles)$$

Rearranging terms allows us to solve for v, as follows:

$$v = \sqrt{[(32.2\ ft/sec^2)(3956.5\ miles)]}$$
$$v = \sqrt{[(32.2\ ft/sec^2)(1mile/5280ft)(3956.5\ miles)]}$$
$$v = 4.91\ miles/sec$$

This, of course, is equatorial velocity at *Condition 0*. We can now determine the length of a "day" at *Condition 0* by dividing the circumference of the earth by equatorial velocity.

$$T = \text{"day"} = 2\pi r/v$$
$$T = \text{"day"} = 2\pi(3956.5\ miles)/(4.91\ miles/sec)$$
$$T = \text{"day"} = 5060.7\ seconds$$
$$T = \text{"day"} \sim 1.41\ hours$$

So, the circuitry must deliver a burst once every 1.41 hours. (Note: This time may be verified by doing an Internet search for "weightlessness at the equator".)
The frequency, f, is given by

$$f = 1/T$$
$$f = (1/1.41Hour)(1Hour/3600s)$$
$$f = 197\ \mu Hz$$

Determining the DC Charging Current

$$v = (1/C)\int idt$$

In this case i will be a constant, I_{charge}, so we can write

$$v = (I_{charge}/C)t$$

$$I_{charge} = vC/t$$

v is known, C is known, and t is known, so we have

$$I_{charge} = (2.5V \cdot 0.001F/1.41Hour)(1Hour/3600s)$$

$$I_{charge} = 0.49\mu A$$

A digital timer is another possible (probably more practical) option for delivering the desired signal.