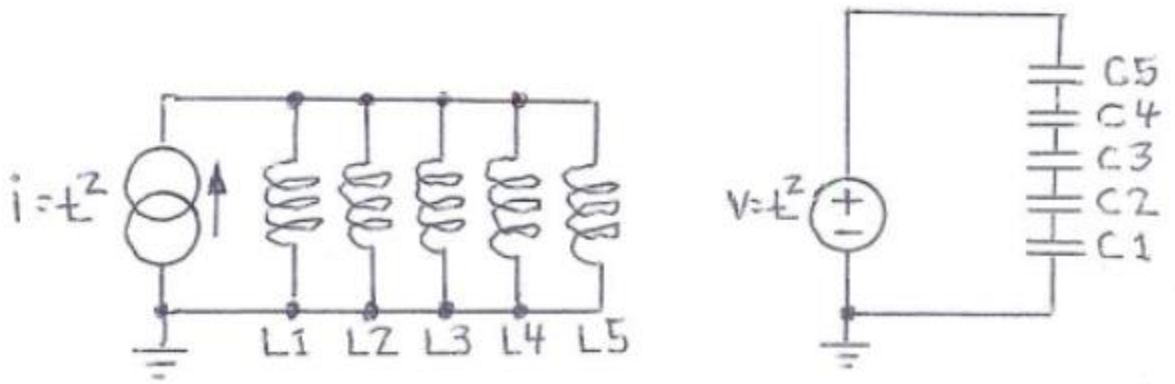
Circuit Challenge 6

Consider the two circuits shown below.



The Situation

James has 5 inductors and a current source. Lisa has 5 capacitors and a voltage source. The inductor and capacitor values are as listed in the table below.

Inductor	Value	Capacitor	Value
L5	8μH	C5	8μF
L4	8μH	C4	8μF
L3	4μH	C3	4μF
L2	2μH	C2	2μF
L1	1μH	C1	1μF

James wants to know when (t=?) 1 Ampere will be flowing in L_1, and he wants to know what the voltage will be across L_1 at that time.

Lisa wants to know when (t=?) 1 Volt will appear on C_1, and she wants to know how much current will be flowing in C_1 at that time.

The Challenge

Determine the times, the voltage, and the current.

Compare your calculations with what you find by scrolling down to the author's analysis.

Analysis for Challenge 6

James' Circuit

L_2, L_3, L_4, and L_5 have a combined parallel inductance of 1µH (equivalent to L_1). Thus, half the input current will always be flowing in L_1, and we can write

$$i_{L1} = t^2/2$$

Then, we solve for t when i_{L1} is 1 Ampere, as follows:

$$1 = t^2/2$$

$$t^2 = 2$$

$$t = \sqrt{2} \text{ seconds}$$

The voltage on L1 is found using the following relationship:

$$v_{L1} = L_1 di/dt$$

For L_1, di/dt = t Amperes/sec. Therefore we can write

$$v_{L1} = L_1 t(A/sec)$$

In this case $t=\sqrt{2}$. Thus,

$$v_{L1} = 1\mu H(\sqrt{2})(A/sec)$$

$$v_{L1} = (\sqrt{2})\mu V \quad \leftarrow \text{ voltage on } L_1 \text{ when the current in } L_1 \text{ is 1 Ampere}$$

Lisa's Circuit

C_2, C_3, C_4, and C_5 have a combined series capacitance of 1µF (equivalent to C_1). Thus, half the input voltage will always appear on C_1, and we can write

$$v_{C1} = t^2/2$$

Then, we solve for t when v_{C1}=1 Volt

$$1 = t^2/2$$

$$t^2 = 2$$

$$t = \sqrt{2} \text{ seconds}$$

The current in C_1 is found using the following relationship:

$$i_{C1} = C_1 dV/dt$$

For C_1, $dV/dt = t$ volts/sec. Therefore we can write

$$i_{C1} = C_1 t(V/sec)$$

In this case $t=\sqrt{2}$. Thus,

$$i_{C1} = 1uF(\sqrt{2})(V/sec)$$

$$i_{C1} = (\sqrt{2})\mu A \quad \leftarrow \text{ current in } C_1 \text{ when the voltage on } C_1 \text{ is 1V}$$

Did You Notice?

Both input sources have finite slopes at t=0.

- If i had instantly stepped from 0 to some non-zero value, $Ldi/dt=\infty$
- If v had instantly stepped from 0 to some non-zero value, $Cdv/dt=\infty$